Reluctant Geometer

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Contents

1 Background

- No geometry course as an undergraduate student
- One geometry course as a graduate student (undergrad./grad. level) - Used Moore Method.
- Doctoral Dissertation - Used geometries to study groups (primarily finite geometries to study finite groups)

2 What Geometry Means to Me

Definition 1. A *pregeometry* over a set I is a set of elements Γ together with a type function t and a reflexive and symmetric incidence relation $\sim$ such that the type function maps Γ onto the set I and such that for any two elements $x, y \in \Gamma$ with $x \sim y$ and $t(x) = t(y)$ we have that $x = y$.

Definition 2. A *flag* in Γ is a set of pairwise-incident elements.

Definition 3. A flag $\mathcal{F}$ is *maximal* if for all $x \in \Gamma$ we have that $\mathcal{F} \cup \{x\}$ is not a flag.

Definition 4. A *geometry* is a pregeometry Γ such that t induces a bijection between any maximal flag of Γ and I.

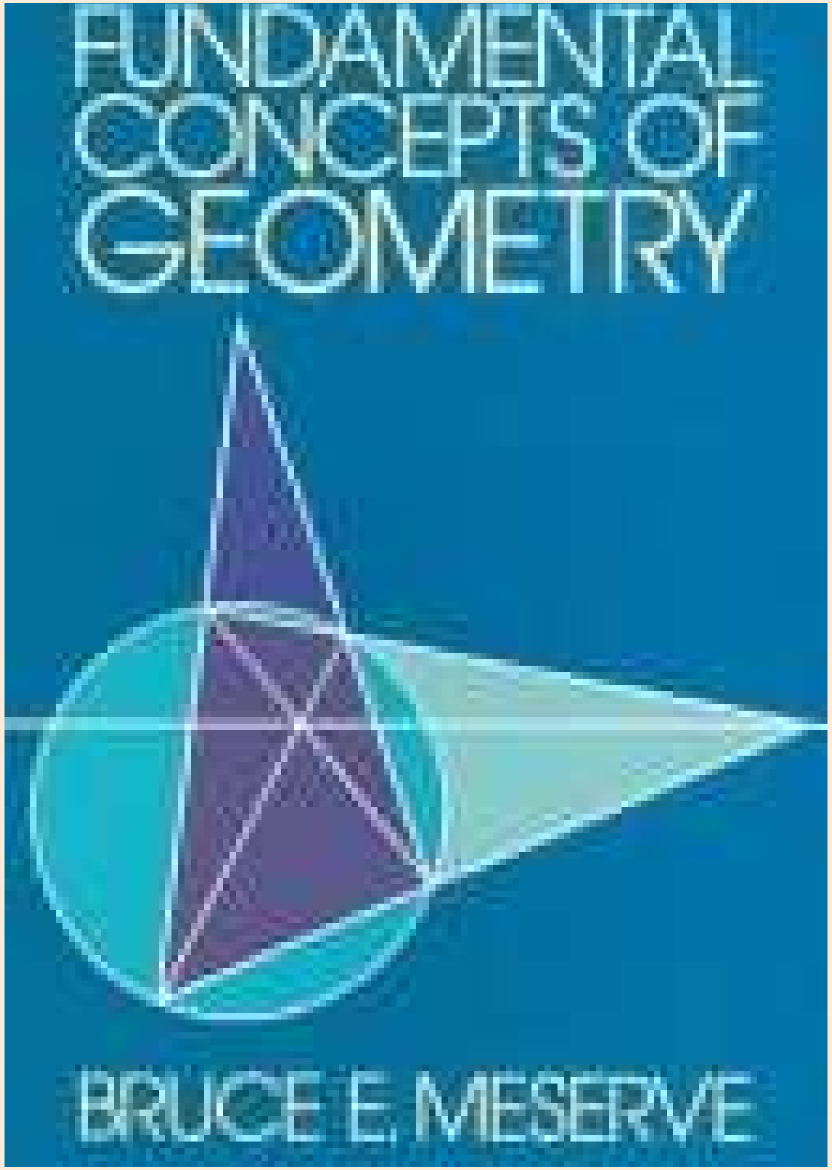
Example 1. If $V = \mathbb{R}^3$ is the real vector space of dimension 3 over $\mathbb{R}$. Then let $\Gamma =$
$\left\{ W \subseteq V \,\middle|\, W \text{ is a subspace of } V \text{ and } \dim W = 1 \text{ or } 2 \right\}$.
Relation is inclusion, 1 dimensional subspaces are points and 2 dimensional subspaces are lines.
This is the real projective plane.

3 Courses

- MATH 357 Modern Geometry - Axiomatic treatment of topics in geometry.
- MAED 557 Geometry/Trigonometry for Secondary Teachers - The course will visit axiom systems, review the core theorems of high school geometry, consider advanced topics in Euclidean Geometry, explore Non-Euclidean Geometries, examine Area and Transformations, look at trigonometry and have student presentations of a relatively new geometric topic. Computer software (Geometer's Sketchpad, Kaliedomania, maple, ...) will be at the center of the students work.

4 What I did - MATH 357

- No preference for a course outline or textbook. Why use a $\$100+$ textbook for which I had no strong opinion?
- Taught projective geometry with the intent to eventually recover Euclidean geometry. I used a Dover book that cost less than $\$10$.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Positives

- Students learned about axiom systems and models.
- They were able to write geometric proofs.
- They learned about projective geometry.

Negatives

- I was not able get far enough to connect projective geometry to the Euclidean geometry the students were familiar with from high school.
- Not able to cover parallel postulate in any depth.
- No history.

5 What I did - MATH 537

- History (axiom systems, Euclidean geometry, historical problems)
- Non-Euclidean geometry (Geometer's Sketchpad)
- Fractal Geometry (Transformations, Matlab)
- Student projects - Sketchpad and a topic from the class

Positives

- Students enjoyed the history and actually solving historical problems.
- Great feedback from students about what would (and would not) be useful for teachers in a high school.
- Both the students and myself learned quite a bit about Geometer's Sketchpad.

Negatives

- None

6 What I learned

- Students do not learn a lot of history in their geometry classes.
- Teachers actually use Sketchpad.
- Students are interested in non-Euclidean geometry.
- Nonstandard topics such as fractal geometry are popular in high school classes.
- Next time in MATH 357 probably use a more standard text such as *Roads to Geometry*.