Localization of Mobile Receivers using Opportunistic Signal Sources; (or)
What to do if your GPS fails

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Class of problems

- Multiple waves initiated from various sources
  - They propagate on a low-dimensional manifold (with boundary), called the *environment*

- Multiple sensors that collect information about the wave signals in their immediate vicinity
  - No synchronization between sensors
  - Sensors may or may not be able to distinguish signals they receive
  - Sensors record a count of received signals and their approximate times-of-arrival
To a good approximation, waves break into three parts:
- Incident wave
- Reflected waves
- Diffracted waves

Thresholding in time or signal strength results in a cellular decomposition of the environment.

These decompositions play a crucial role in our methods.
Simultaneous localization and mapping

This class of problems is part of a larger framework:

From data collected from various sensors, we answer two kinds of questions:

- **Mapping**: what is the topology or geometry of the environment?
- **Localization**: where are the sensors within their environment?

If this can be done *simultaneously*, so much the better
Opportunistic emphasis

We have a strong preference for

- Passive sensors, which are not also signal sources
- Minimal communication between sensors
- Uncontrolled sources: We don't know their:
  - Location
  - Time of initiation
  - Power level
  - Preferred directions
Opportunistic emphasis (cont'd)

Why is this useful?

- No need to spend effort placing or measuring source locations
  - This is a major source of error in radar systems
- Can operate transparently
  - No “pollution” of the radio spectrum
  - No need to coordinate with the operators of the sources
- Plenty of sources already available
Standard methodology

This is typically viewed as a tomography problem

- Single (moving) source
- Small number of sensors (possibly also moving)
- Careful synchronization, measurement of source and sensor positions required
- Integral transform methods used extensively
- Environment has trivial topology

Computational explosion when many uncontrolled sources are used, or if the environment is complicated
Lightning-based localization

- Sources are lightning strikes
- Fixed, carefully surveyed network of sensors, which may communicate
- An additional collection of mobile sensors, that cannot communicate
- Task: figure out where the mobile sensors were after they've been recovered
Lightning facts

- There are roughly 45 lightning strikes per second over the entire surface of the earth.
- The radio signature of lightning can be detected at a distance of 10,000 km (pole-to-equator).
- Radio signature for a given lightning strike is essentially unique.

The v2.2 gridded satellite lightning data were produced by the NASA LIS/OTD Science Team (Principal Investigator, Dr. Hugh J. Christian, NASA / Marshall Space Flight Center) and are available from the Global Hydrology Resource Center (http://ghrc.msfc.nasa.gov).
Why might this be useful?

- Provide accurate tracking of an object in regions where GPS is unavailable
  - Underground
  - Underwater
  - Within dense cities
  - Where the GPS receiver causes too much interference

- When mobile sensor leaves this GPS-denied region, it can be recovered
A solution

- Strikes are infrequent enough that mobile can be synchronized after it's been recovered
- Recover the lightning ordering as a result (lower bound on accuracy)
- Short time difference resolution sets accuracy of localization
Induced cell complex

- The mobile sensor records only the time differences between successive lightning strikes
  - The time differences are quantized into “short”/“long” or some similar small set of values
  - This yields a data rate on the order of megabytes per hour
- With accurate location and timing information at the fixed sensor locations, we form a cellular partition of the region between them
  - We can propagate the lightning pulses into the region precisely
  - Each cell has a label, which is assigned by the sequence of lightning strikes in the order they're received (or if several arrive simultaneously)
  - With high probability, these labels on the cells are unique, and localize the mobile sensor coarsely
  - Localization accuracy is optimized by looking for short time differences within the received signal (they are the high codimension cells)
Poisson-Voronoi analysis

What is the typical cell radius after \( N \) sources have been received? (Coarse localization accuracy)

- This question can be answered for randomly-distributed sources in the plane (\( \lambda \) is the density of sources):
  \[
  \frac{2}{3 \sqrt{\lambda}}
  \]

- It's likely an open problem to recompute this for the sphere, though it's largely tedious and uninspiring

<table>
<thead>
<tr>
<th>Time Elapsed (s)</th>
<th>Number of Strikes</th>
<th>Cell size (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>2700</td>
<td>281.98</td>
</tr>
<tr>
<td>3600</td>
<td>162000</td>
<td>36.40</td>
</tr>
<tr>
<td>36000</td>
<td>1620000</td>
<td>11.51</td>
</tr>
</tbody>
</table>

Worst-case average cell size. A more useful figure is *expected* cell size, which is not as easy to compute.
Numerical simulation

- Rather than computing typical cell radius analytically, we can use a numerical simulation.
- This also has the benefit that we can limit the visible strikes to the ones within 10,000 km.
- Additionally, we then have access to the full distribution of cell sizes.

White boxes are source locations.
Why it works: Uniqueness of channel response

- Underlying the solution to the lightning localization problem is a deeper fact
- The signal profiles received at every given point in the environment are *unique*, given enough sources
Uniqueness of channel response

- Given a sufficiently large number $n$ of randomly distributed signal sources in an environment $E$, the channel impulse response map $C:E \rightarrow (L^2(R^+))^n$ is a topological embedding.

- The channel impulse response map records the received signal strength as a function of time for each source at every point in the environment.
Uniqueness: Applications

This idea underlies all imaging methodologies:
- Human vision (using angular diversity and parallax)
- Human hearing (parallax)
- Radar imaging (using timing and doppler shift)

But better than one might suspect, the uniqueness works even if the domain has complicated topology
- Reflections, diffractions, etc. are useful!
  - They improve one's ability to distinguish nearby points
A more complicated problem

- Urban propagation, where the sources are man-made broadcast signals
  - Cell phone services
  - TV, radio stations
  - Others
- Sensors may not be well-surveyed
Why traditional methods fail

- Computational overload due to “multipath”
  - Reflected signals are very hard to track or predict
  - These need to be carefully removed to make integral transform methods work
- Statistical methods don't have the resolution needed to provide good mapping capability
Some applications: mapping

Use the sensor network to measure the topology and geometry of the environment

- Topology: Use the regions visible to each sensor as a cover
  - Common signals heard by sensors imply overlapping coverage regions
  - Suitable disambiguation using the uniqueness theorem is necessary
- Geometry: Uses time-difference-of-arrival (TDOA) between echoes
  - Not so easy to reconstruct the geometry from measurements
Some applications: motion planning

- Looking at TDOA can alert a sensor when it is getting close to an obstacle
- Once a map of the environment has been constructed, future sources or sensors can be localized within this map
  - Regions of poor accuracy can be identified
  - More sensors can be directed into these regions
Visibility as a means for measuring topology

- We want to make use of coverage regions of each sensor to extract the topology of the environment
  - Assume that these regions form a cover for the environment
  - Use the *nerve* construction
Disambiguation of intersections

- If the mutual intersections of the elements of a cover are each contractible, then the nerve has the same homotopy type as the environment. (The Nerve Lemma)
- In general, coverage regions will not have connected (let alone contractible) intersections!
- So the intersections need to be disambiguated somehow
Signal response disambiguation

- One might disambiguate using a directional antenna, but this goes bad after a few reflections.
- Uniqueness of channel response gives a more robust way: sources in different components of intersections of coverage regions will have very different received signal profiles.
  - A clustering algorithm should be able to tell the components apart, since the channel response map is continuous.
Extracting geometry

  - This is an old, hard problem!
  - It's worse if the topology is complicated
- But by working with a discrete variant (with a finite collection of sources and sensors), we might be able to make some headway
Computational limitations of geometry extraction

- A brute-force approach might be a good way to do this if
  - Topology of the environment is trivial
  - Sensor and source locations are known to a high degree of accuracy
  - This is essentially the same problem as radar image formation
- But brute force isn't going to work if the topology is complicated
  - How do you know which regions interact?
  - There are many unknowns defining the geometry of each obstacle
Visibility sheaves

- Essentially, what we're trying to do is assemble *local* measurements at each sensor into a *global* picture of the environment.

- The usual tool for storing local data for such an assembly is a *sheaf*.

- Briefly, a *sheaf* is a gadget that assigns some kind of (usually algebraic) data to each open set of a space, subject to the fact that overlapping regions must agree on their intersection.

- Sheaves are usually merely *convenient*.
  - In all cases I'm aware of, they aren't necessary.
Visibility sheaves (cont'd)

- A visibility sheaf captures signal responses in its algebraic data, but also keeps track of which sensors can see this data.
- More precisely, a visibility sheaf assigns to each open set $U$ the module of continuous functions taking values in $R^n$, where $n$ is the maximum number of sources visible to points in $U$.
- Note: Visibility sheaves are a special case of a construction of Swan (1957), though he used them differently.
Visibility as a computational reduction

- Using the visibility data, (stored conveniently in a visibility sheaf) we can partition the geometry problem into numerous smaller \textit{canonical} problems.
  - Each canonical problem involves a small number of unknowns and some relations between them.
  - However, each canonical problem is \textit{under}determined.
  - The missing relations connect neighboring canonical problems.
  - This suggests the possibility of distributed computation.
Reduction of freedom via topology

- Essentially, we'll arrive at a banded, nonlinear system, which is easier to solve than the full system. However, we might be able to do better!

- In many local-to-global settings, topology provides constraints that rule out certain possible solutions:
  - A good example is the theory of smooth vector fields: on the sphere there must exist at least two zeros.
  - This is called an *obstruction theory*. One might exist for visibility sheaves of geometric information.
Summary

- We have discussed a variety of discrete topological imaging problems and why they might be useful.
- The key insight is the uniqueness of channel response, which underlies all imaging.
- This allows us to formulate a procedure for extracting the topology of the environment.
- In simple situations (the lightning problem), geometric information is easy to extract.
- In more general settings, the extraction of geometry is hard.
Future directions

- Estimate the average cell radius on the sphere using actual lightning frequencies
- Examine the general geometry extraction problem in detail
  - Look for an obstruction theory
  - Discern geometry models that yield computationally-efficient canonical problems
- Computational questions
  - How much of the topology or geometry extraction can be computed in a distributed fashion?
  - How can we minimize interprocess communication?