Mathematical Learning

for the Economic and Finance Student

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In as friendly and useful manner as possible, my goal is to provide an overview of the specific mathematics most useful in undergraduate Economics and Finance courses. The math department introduces many of these math concepts. The hope is that presenting the math department certain math tools expected and required the math department can develop these concepts sooner. Students exposed to these math concepts sooner become more connected with the economics presented and perform better in the course. Consequently, Economics is concerned with human behavior, cause, effects, actions, and reactions. We can talk Economics and learn Economics but performing Economic and Financial Analysis requires math. The universal goal is to increase value added to our student graduates. Putting better math into Economics and Finance will help us accomplish this goal

Introduction

This presentation is designed to set out some of the basic mathematical concepts needed for students in Business and Economic. While the Math Department clearly teaches these concepts, our goal of sharing and discussing a systematic guide on the math most effectively utilized by Finance and Economics will benefit all parties.

Principles of Economics are generally taught in a non-mathematical way with very limited references to math. However, basic math concepts are necessary even in Econ-principles. Initially

economics can be presented in a graphical and qualitative form of analysis. For instance, the demand for a normal good falls if household income declines. This is a qualitative statement and visual statement when shown on a graph. Business and Finance decisions require a quantitative statement expressed by utilizing mathematical models to give concrete expressions to this theory. Higher level business, Finance and Economic courses rely on the hands on application of math in the analysis of a business decision. Graphical presentation is not enough.

The problem we face is math anxiety in that requiring specific levels of math results in a loss of majors and a higher failure rate with loss of retention in the major. On the other hand, the value of out product (Business and Economic graduates) is greatly enhanced when rigorous math skills are applied within these majors.

Years of research in education suggest that students develop individually specific learning styles (Felder, 1996), (Dunn and Dunn, 1993). Consequently, problem solving is best accomplished with a strategy-building approach (Eyring, Johnson, and Francis, 1993). In the best interest of both departments I believe that progress can be developed if we apply specific Kolb's models of learning to math for Economic and Finance. (See Evans, et al., 1998, for a discussion of Kolb's Model). In the Kolb model there are four basic types of learners:

A. Concert, reflective: those who build on previous experience.
B. Concrete, active: those who learn by trial and error.
C. Abstract, reflective: those who learn from detailed explanations
D. Abstract, active: those who learn by developing individual strategies

Consequently, I believe, subject to open minded discussion and criticism that learning process A, and C may be most beneficial for learning math required for Economics and Finance applications. This methodology when addressed by our departments should develop a positive connection between Math and Finance and Economics and at the same time reduce the problem of "memorize and regurgitating" the current class objective and "math anxiety associated with these majors. Financial and Economical analysis cannot be memorized. (Note this line of reasoning is developed following Knisley, Jeff "A Four-Stage Model of Mathematical Learning").

This paper will now develop a basic outline of math tools required for specific courses in Finance, Economics and Business with learning method A above emphasized. By developing specific math requirements early, the students learning process in Finance, QBA and Economics is enhanced by the concrete, reflective process of building on previous experiences. Early introduction to Finance math, calculus for Economics and statistics specific to Quantitative Business Analysis at an introductory level will develop a prior level of experience that can make the learning process easier and more successful. We want the student exposed to this math as soon as possible. The results is a value enhanced business graduate, less math anxiety, better retention of business majors and growth for the math department and the business department.

Many students at Mansfield and other Universities may start undergraduate courses with less than A-level mathematics which results in reduced mathematical requirements for some advanced level courses. Students with little mathematical knowledge can still learn to apply simple math techniques such as present value and future value and other powerful math concepts to develop an analysis. Many students are convinced they have little math skills which perpetuates their low math confidence. Some of

this confusion may be related to math intuition and not necessarily their individual capacity to learn simple math techniques. Students should be convinced that they can solve the problem and should know when an appropriate solution is developed and develop confidence over time. Problem solving abilities develop slowly over time. Student confidence and ability comes from continuous experiences in solving problems in varying context and different levels of complexity (Kantowski, 1981).

In Economics I often find that students can solve most-one-step problems but run into difficulty when a problem has multiply steps (Carpenter et al., 1981). Consequently, math teachers may help business students improve as problem solvers if they are willing to ask more questions concerning Economics and Business problems and use problem resources other than the mathematical textbook (Suydam,, 1987) Other resources can be developed from Economics, Finance, Operations Management, and Quantative Business Analysis text. Teachers can gain success by choosing instructional activities that integrate everyday uses of mathematics into the classroom learning process (Fong et al., 1986).

Principles of Economics:

Principles of Microeconomics or Macroeconomics are often taught to a large number of first year students. These students are expected to have a strong understanding of basic mathematical operations in addition, subtraction, multiplication and division and hopefully some competence in Algebra. Students weak in Algebra are encouraged to take College Algebra course. Potential areas of weakness tend to become apparent when students are asked to graph two variables of a functional relationship between Y and X Economics logic often suggest that two variables are linked in a specific way. We have positive relationships and negative relationships between two variables. A demand curve has an inverse relationship between price and quantity demanded. While the supply curve has a direct relationship between price an quantity supplied.

Concept of Linear Functions Students need to recognizing the intercept and slope when presented, understand the concept of slope and a positive or negative relationship between the two variables. For instance, how much the quantity demanded of pizza changes with an increase in the price of pizza. Students should have no difficulty understanding that the slope of the line is positive for lines that are upward sloping and negative for line that are downward sloping. A slope of zero is a horizontal line. The slope often measures the tradeoff between two variables in a production function or may represent a measures of the opportunity cost or tradeoff. The relative price ratio of two goods is often determined by the slope of a Productions Possibility Frontier.

Market equilibrium exists where the demand and supply curves intersect. At that point P* is the equilibrium price and Qd*=Qs* quantity demanded is equal to the quantity supplied. Student clearly understand the equilibrium on the graph but some have difficult when ask to solve a simple simultaneous equation of supply and demand for the equilibrium price and equilibrium quantity. Such as: Qd = -2P + 20 and Qs = 2P – 4. Students fail to associate the equations with the expressed economic relationship clearly displayed in a standard supply and demand graph. Finally, at the Principal level in Economics students are introduced to the concept of Marginal Analysis where the optimal decision is to continue any activity up to the point where the marginal benefit equals the marginal cost– in symbols MB = MC. While simple in concept, marginal analysis is a powerful tool that illuminates many central aspects of decision making regarding resource allocation. While non-linear equations may be a step for new students the slope of a nonlinear curve as measured by the slope of the tangent line has powerful implication in understanding MB and MC. For instance, a total profit function will reach a peak and then

turn lower. Here students are encouraged to look at incremental changes or marginal changes. Thus the marginal value of profit (marginal profit) is the change in total profit associated with a 1-unit change in output. The important implication about the marginal relationship here is that total profits are maximized when the marginal value shifts from positive to negative. Total profits are maximized where the slope of the tangent line is zero. This is a powerful visual concept and extremely important for Business and Economic students who continue into upper level courses.

A summary of math related skills for the typical Principals of Micro or Macroeconomics:

a. Basic math operations
b. Concepts of fractions, percentages, and the use of powers
c. **Basic rules of Algebra**
d. **Concepts of graphing and coordinates**
e. **Equations of a straight line: intercept and slope**
f. **Simple relationships and complex relationship between two variables**
g. **Solving simple simultaneous equations**
h. **Slope of non-linear relationships and the relationship between marginal and average values**

The bold faced type is the primary area of mathematical development in Principles.

Intermediate and Managerial Economics and other upper level Economics courses.

Lets assume that continuing economic and business students have successfully progressed through the basic math associated with the Principles of Economics and want to begin higher level business and economic courses. Clearly the mathematical prerequisites need is a one-term introduction to calculus as applied to business, economics. I strongly feel this is the best approach to producing value added to our graduates in Economics and Business. The student should learn to do economics by actually applying the mathematical tools to problems in economic theory and this is most easily accomplished with a minimum of Differential Calculus. Students become more connected to intermediate economic when economic applications produce quantitative results.

Intermediate economics requires the use linear and non-linear equations and the ability to interpret economic behavior from them. More importantly we move the focus away from linearity and utilize more non-linear equations in out models which can make material more difficult when only using algebra and easier with minimum calculus. Student can more easily grasp non-linear equations when presented a dynamic visual on excel worksheet with given values for x and changing the values of a, b and c. (METAL). Students should be familiar with the Quadratic Formula and cubic and other polynomial functions. However, most economics generally relies on linear or the quadratic function with little use for higher order functions.

A typical function may be a total cost function for a perfectly competitive firm. To determine profit maximizing output level one sets the marginal cost MC = MR marginal revenue. and Price is = $30 and since this is a perfectly competitive firm P = MR= $30. Setting = 30 Q = 6 is the optimal output level. However, the price must be above the average variable cost curve or the firm should shut down. Since total variable cost = its average variable cost is equal and since $18 is the AVC the firm will produce 6 units. Another problem area for students is in understanding the concepts of elasticity which is very difficult to explain in a visual concept. The price elasticity of demand is a measure of how sensitivity quantity demanded is to a change in price. The price elasticity demand is defined to be the percentage change in quantity demanded resulting from a 1 percent change in price. Consequently with out basic differential analysis explaining to students that price elasticity of demand varies along a linear demand curve conflict with the students visual concept of a linear demand curve and slope. Student confuse slope with price elasticity of demand which is wrong.

Differential calculus is the economist tool for marginal analysis. Differential calculus in economics has a long history with many of the formal ideas developed by the mathematician's readily absorbed into the tools and language of economics. Students must cover introductory concepts in differentiation through constrained optimization. The mechanics and rules of differentiation may not be too difficult for students to grasp. However, many students will not develop any significant understanding of limits and will survive by the rules alone. These rules can give students the ability to complete simple differentiation with out the need to fully understand the concept and this is OK. Here it is: "studies have shown that almost no one completes a first calculus course with any meaningful understanding of limits" (Kinisley,) (Szydlik, 2000). Consequently, instructors need to make the connection to students that the rate of change is important to economics and marginal analysis and that differentiation is a fundamental tool in economics and that differentiation can be learned. This can be accomplished by practice and step modeling for solutions. Business students need cook-book calculus. They need to learn the rules to differentiate a single variable function and partial differentiation. I am sure that the Product and the Quotients rules may be difficult at first but with repetition student can learn to follow these rules of differentiation. Business student do not necessarily need to complete proofs. I understand the need to develop a formal definition of the derivative of a function in terms of limits but this section could be minimized so students can get to the rules. Integration while important is only used at a minimal level to measure consumer and producer surplus and this can be accomplished with areas of rectangle and triangle. Students should be introduced to bi-variate functions and the concept of partial differentiation. Constrained optimization can be developed in the Intermediate Microeconomics class. Finally using other problem resources may be helpful so the student has a number of related problems to practice.

A summary of math related skills for Intermediate and Higher Level Economics

a. Slope concept of slopes
b. Equation of straight line
c. **Algebra**
d. **Quadratic Functions and the Quadratic Formula**
e. **Simultaneous Equations**
f. **Differentiation of single variable functions (Rules Based)**

g. **Concept of Maximization and Minimization and Second Derivative**
h. **Concept of Bi-Variate Functions**
i. **Partial Differentiation**

Finance and Investment Theory

I have been fortunate in that I have had the opportunity to teach Personal Finance, Introduction to Finance, Corporate and Managerial Finance, and Modern Portfolio Theory. Finance requires a lot of hands-on application in that you learn by doing. We often crunch the numbers by hand to get student familiar with the concept and than utilize Excel for larger problems. Excel is an essential tool for teaching finance. But Excel requires that one understands the mathematics behind the finance. So what math is utilized in Finance?

Finance usually begins with the Time Value of Money concepts and simple and compound interest.

In Personal Finance and Intro-to-Finance, I encourage students to learn this equation and not to rely on tables. It is important to show students how to apply present value and future value tables and than convince them that a simple calculator is easier. How often do you find someone with a set of tables when you need them? I show students how easy it is to calculate with a basic calculator $10 from Wal-Mart and how different table values are determined. Consequently, most students elect to use a simple calculator in calculating time value. What is important with this approach over that of using a business calculator initially is that students must understand how to set up the problem to fit the equation. Students need to determine if it is a FV or PV problem. If the question requires solving for a PV a simple manipulation of the above equation is all the student need to understand to come up with . Next I present the calculations for compound interest as a simple adjustment to the standard equation

I encourage students to remember the standard equation and to remember to adjust this equation when multiply compounding periods are required. For annuities I also encourage student to utilize PVA and FVA equations and initially, math anxiety occurs initially until students are show through a number of problems that the equations fairly simple to use again with a basic calculator.

FVA = *and the PVA* = With multiply compounding periods again I explain the need to adjust the interest rate and the number of periods. Practice with a number of problems produce good retention.

Time value development precedes capital budgeting techniques utilizing NPV rule and the IRR internal rate of return rule. Time value is important in determining the value of an asset. The value of most financial asset are based the present value of all perceived future cash flow.

The Arithmetic Average and the Geometric Average is utilized often in finance with the Geometric Average often used to estimate the growth rate of dividends or earnings over a given time period. Risk and risk premiums are developed through the use of a probability scenario and probability distribution. Students are expected to know how to calculate the expected return or the mean value of a distribution of

holding period returns and the variance and standard deviation of a single security. For students to properly understand efficient diversification students need to understand covariance and correlation and how the correlation coefficient is calculated. In option valuations, the Black-Scholes Formula uses natural logarithm and probabilities drawn from a standard normal distribution.

A summary of math related skills for Finance and Investment:

a. **Reinforcement of Algebra**
b. **Simple and Compound Interest**
c. **Time Value and rules for NPV and IRR**
d. **Geometric Averages and Arithmetic Average**
e. **Exponential and logarithmic Functions**
f. **Expected return, standard deviations and variance**
g. **Intro to Probability distribution, discrete and continuous**
h. **Covariance and correlation coefficient**
i. **The Normal Distribution and Central Limit Theory**

Business Statistics:

It is important for students to understand the relevance of statistics. The Business and Economics department needs an introductory course in Business Statistics which provides the tools and techniques that are used to convert simple data into meaningful information in which business decisions can be made. The math department provides courses in basic statistics to satisfy a variety of major. However, the best frame of reference for business and economic students should be the functional areas of business. These areas are accounting, economics and finance, information systems, production and management, and marketing. Applications of statistical topics need to be in an applied context related to one of these areas. I am sure that general statistic does find a number of applications in these areas. However, students may have difficulty in determining the specific application and interpretation of results. Therefore, if general statistic courses can be biased more in the direction of application to business I believe both departments will benefit.

Students should use software that is typical in the business world. This is software that one finds in Excel spreadsheets and other basic statistical software readily available. Students should have exposure to different software for statistical analysis to develop a learning curve with variety of experience. However, statistics must be also be-hands-on, calculated by plug and chug by hand or with a calculator. This gives students a feel for the data a get your hands dirty application. QBA, Finance are hands on applications, student work out a large number of problems by hand and by individual programming in Excel. Hand-on calculations develop a higher level of understanding of the specific concept. Practice is important and in class group exercises and homework that involve actual or realistic

data develop a stronger association with the application and statistical theory.

What can the math department do to implement these strategies? Send for some Business Statistics text book and utilize their data base and question resources. There are a number of texts for a First Course in Business Statistics available. On the other hand, it is possible that many departments have moved in this direction and have interrogated more business application into their general statistic course. If so, than it is important to ask for feedback from the business and economics department to access any benefit or problems. Also, proofs are not necessary for business students to master statistics. To prepare for upper level QBA and Finance courses students need introductory exposure to probability, discrete probability distribution (binomial and Poisson) and continuous probability distribution, the normal distribution. I am sure that these areas are covered in a basic statistics course. In the following section I will present a general outline of statistic to prepare business students for higher level Quantitative Business Statistics Remember what is important in the math department course criteria is specific to your field and statistics is your field. The Business and Finance department desire a mutual beneficial outcome for all involved.

A summary of math related skills for Business Statistics:

a. Presenting Data in Tables and Charts: collection and types of data
b. Numerical Descriptive Measures: descriptive statistics
c. Basic Probability Concepts and Conditional Probability
d. Probability Distributions for a Discrete Random Variable
e. The Normal Distribution and Evaluating Normality
f. Sampling Distributions of the Mean, Proportion, and Sampling Methods
g. Confidence Interval Estimation: (Known and Unknown Means)
h. Hypothesis-Testing: One Sample Test: Z Test, One-Tail, P Value, T-test
i. Intro to Regression Analysis

(Levine, Krehbiel, and Berenson. "Business Statistics A First Course 4th ed.)

Statistic involves an enormous amount of material and work and students of all ages have a difficult time understanding and learning statistics. A reflection back on the above material I am sure that many of these concepts are covered over the semester in a variety of courses. Business and Economics desire a strong math base to create enhanced value in our graduates. The problem is that we need exposure to Business Calculus and Statistic as early as possible for our students. However, many students lack good Algebra skills making the study of both Business Calculus and Statistics more difficult. I would suggest that consideration be giving to requiring all Business and Economic students to have exposure in an Algebra course their first semester and than either Business Statistic or Calculus in the second and third semester. However, math anxiety results in a decline in majors a problem we all face. On the other hand, Quality business schools require that all Business and Economic majors take courses in Business Calculus and Statistics. I believe if we strengthen new students algebra skills we may have more success in both statistic and calculus. I have presented a brief overview of the types of math Business and Economics students

need exposure to.

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